Research statement

The results obtained during the work on my thesis and in the years following its defense are in the field of model theory, and its applications to algebraic geometry. More recently, I have taken an interest in non-Archimedean geometry and motivic integration. I was inspired to start working in this field (and in particular on Project 2a below) after learning in a conversation with Ehud Hrushovski about an arguably useful and underappreciated tool, the asymptotic valued field, defined by Abraham Robinson using ultraproducts of real closed fields, that allows one to relate collapsed Gromov-Hausdorff limits to the geometry of non-Archimedean analytic spaces.

I will quickly summarize my work in model theory and then proceed to describing my current research projects.

1. In a joint paper with Assaf Hasson [HS17] we settled the Restricted Trichotomy conjecture of Boris Zilber in dimension 1. This conjecture plays a crucial role in Zilber's model-theoretic proof [Zil14] of a strengthening of Bogomolov's Torelli-type theorem for curves over finite fields [BKT10]. Our theorem can be regarded as an analogue of the Fundamental Theorem of abstract projective geometry [Art57, Har67].

2. In papers [Sus16] and [SSZ14] (joint work with Boris Zilber and Vinesh Solanki) I studied certain classes of model-theoretic structures called Zariski geometries and developed a criterion for them being representable using Zariski closed subsets of Cartesian powers of algebraic varieties.

Project 1. Refined tropical multiplicities and motivic integraton.

Let Δ be a lattice polygon, and consider the projective toric surface $Y(\Delta)$ associated to it. Let $|L(\Delta)|$ be the linear system containing all curves with Newton polygon Δ. The *toric Severi degree* $n^{\Delta,\delta}$ is the number of integral curves in the linear system $|L(\Delta)|$ that have δ nodes and that pass through $n-\delta$ points in general position, where n is the dimension of the linear system $|L(\Delta)|$. Denote by g the genus of the general smooth element of $L(\Delta)$. According to an idea proposed by Yau and Zaslow, and developed by Beauville, Fantechi, Göttsche and van Straten [Bea99], [FGvS99], if $C \to T$ is a family of curves on a smooth surface where only finitely many rational curves occur, and if these curves have only nodal singularities while the rest of the fibres are smooth, the number of curves with g nodes can be computed as the Euler characteristic of the relative compactified Jacobian $\mathrm{Jac}(C/T)$ of C over T.

This technique can be generalized to the case of strictly positive cogenus $g-\delta$; provided that the family C has finitely many curves with δ nodes and all other fibres are of geometric genus greater than $g-\delta$, one can define numbers $n_1,\dots,n_\delta$ by performing a variable change on the generating series of Euler characteristics of the relative Hilbert scheme of point $\mathrm{Hilb}^{[i]}(C/T)$, and $n^{\Delta,\delta}=n_\delta$ [KST11], [PT10]. In [GS14] Göttsche and Shende have proposed to replace the Euler characteristic with Hirzebruch's χ_y-genus in this construction. They show that the generating series mentioned above give rise to a well-defined *refined count* $N_\delta \in \mathbb{Z}[y,y^{-1}]$.

Both toric Severi degrees and refined toric Severi degrees can be computed tropically as a sum over tropical curves $\Gamma \subset \mathbb{R}^2$ of degree Δ and genus $g-\delta$ passing through $n-\delta$ points in general position of (refined) *tropical multiplicities* $N(\Gamma)$ of Γ, the latter is computed combinatorially in terms of the shape of the graph Γ. In [NPS16] Nicaise, Payne and Schroeter propose a geometric interepretation of the refined tropical multiplicities of Block and Göttsche using the motivic volume of Hrushovski and Kazhdan [HK06]. Motivic volume is a map $\mathrm{Vol}: K_0(\mathrm{VF}) \to K_0(\mathrm{Var}_k)$ from the Grothendieck ring of non-Archimedean semi-algebraic subsets of algebraic varieties defined over an algebraically closed valued field to the Grothendieck ring of varieties over the residue field, and is characterized by the following two properties:

1. if $\Delta \subset \mathbb{R}^n$ is a polytope then $\mathrm{Vol}(\mathrm{trop}^{-1}(\Delta)) = \chi'(\Delta)(\mathbb{L}-1)^{\dim\Delta}$ where trop is the *tropicalization map* (coordinate-wise application of $\log|\cdot|$), $\mathbb{L}$ is the class of the affine line, χ' is the modified Euler characteristic $\chi'(\Delta) = \lim_{a\to\infty}\chi_c(X \cap [-a,a]^n)$ and χ_c is the Euler characteristic with compact supports;

2. let $\mathscr{X}$ be a flat projective scheme over the value ring R such that $\mathscr{X} \otimes_R K \cong X$ and let $\mathrm{sp}: X(K) \to \mathscr{X}_0$ be the specialization map. If $U \subset \mathscr{X}_0$ is a constructible subset then $\mathrm{Vol}(\mathrm{sp}^{-1}(U)) = [U]$.

Additive invariants of varieties, such as the Euler characteristic and the χ_y-genus, can be lifted to $K_0(\mathrm{VF})$ by evaluating them on the motivic volume of the semi-algebraic set. Let S be a set of $n-\delta$ points on the dense torus in $Y(\Delta)$ in general position. If $\Gamma \subset \mathbb{R}^2$ is a tropical curve then denote $C_\Gamma \to T_\Gamma$ the universal family of curves on $\mathbb{P}^2$ of degree Δ and passing through S that tropicalize to Γ. Nicaise, Payne and Schroeter make the following Conjectures [NPS16] that they check for $g=1$:

Conjecture 1. Assume that $\delta = g$ and let $\Gamma \subset \mathbb{R}^2$ be a rational tropical curve of degree Δ passing through points in $\mathrm{trop}(S)$. Then the refined tropical multiplicity $N(\Gamma)$ is equal to $y^{-g}\chi_y(\mathrm{Vol}(\mathrm{Jac}(C_\Gamma/T_\Gamma)))$.

Conjecture 2. For any integer $\delta \in [0,g]$, let Γ be a tropical curve in $\mathbb{R}^2$ of genus $g-\delta$ and degree Δ passing through $\mathrm{trop}(S)$. Then the refined tropical multiplicity $N(\Gamma)$ is equal to $y^{-\delta}N_\delta(C_\Gamma)$ where $N_\delta(C_\Gamma)$ is computed from the generating series similarly to $N_\delta(C)$.

In view of Conjecture 1 it is natural to try to compute the motivic volume of semi-algebraic families of Jacobians of smooth projective curves. According to computations of Nicaise, Payne and Schroeter, the χ_y-genus of the motivic volume of a semi-algebraic family of varieties can be non-zero even if the χ_y-genus of each fiber vanishes (as is the case for Abelian varieties, for example). Nevertheless, I have found the following

Theorem (S.[Sus18b]). The motivic volume of a semi-algebraic family of Abelian varieties that admits a fibrewise uniformization by an algebraic torus vanishes.

My **next goals** in this project are as follows. Firstly, I intend to prove an analogue of the theorem above for relative Hilbert schemes of points of families of Mumford curves (that is, curves that admit a rigid-analytic uniformization). Secondly, I am going to find a formula for the χ_y-genus of a semi-algebraic subfamily of the relative Jacobian of a one-dimensional family of curves in terms of its monodromy, and similarly for semi-algebraic subfamilies of the relative Hilbert schemes of points. Finally, I will study the relation between the χ_y-genus of such semi-algebraic sets and the tropical multiplicity $N(\Gamma)$.

Project 2. Degenerations of complex manifolds and non-Archimedean geometry

2a. Non-Archimedean limits of differential forms, Gromov-Hausdorff limits and SYZ conjecture.

In the beginning of the 2000s Kontsevich and Soibelmanhave come up with a new approach to the Ströminger-Yau-Zaslow (SYZ) conjecture from the subject of mirror symmetry [KS04]. In a nutshell, this conjecture posits that two mirror partners X and $X^\vee$ admit two fibrations over the same base B, a manifold of real dimension $1/2\dim_{\mathbb{R}} X$, and that generic fibres of the two fibrations are tori that are dual to each other. Kontsevich and Soibelman suggest to look for the base of the SYZ fibration using two methods, a non-Archimedean and a differential-geometric one.

Let $\mathcal{X} \to \mathbb{C}^\times$ be a family of projective complex manifolds with maximal unipotent monodromy around 0 and let $\mathcal{L}$ be a relatively ample line bundle on $\mathcal{X}/\mathbb{C}^\times$. Let $X = \mathcal{X} \otimes \mathbb{C}((t))$ with respect to the natural inclusion $\mathbb{C}[t] \hookrightarrow \mathbb{C}((t))$ and let L be the restriction of $\mathcal{L}$ to X. Denote $\mathcal{X}_s$ the fibre over a point $s \in \mathbb{C}^\times$ and assume that $\{\omega_s\}_{s\in\mathbb{C}^\times}$ is a smooth family of Kähler forms of Ricci-flat metrics on $\mathcal{X}_s$ for s in a punctured neighbourhood of 0, such that $\omega_s \in c_1(\mathcal{L}_s)$. Let $\tilde{\omega}_s$ be the Kähler form of the metric normalized so that the diameter of $\mathcal{X}_s$ is 1.

On the one hand the base of the SYZ fibration is to be identified with a subset, called *essential skeleton* $\mathrm{Sk}(X)$, of the Berkovich analytification X^{an} of the variety X, and on the other hand it is to be constructed as a Gromov-Hausdorff limit B of $(\mathcal{X}_s, \tilde{\omega}_s)$ as $s \to 0$ ("large complex structure limit").

Conjecture A (Kontsevich and Soibelman, Gross and Wilson, Todorov) The limit space B contains an open dense subset B^{sm} which has the structure of an affine manifold, $\dim_{\mathbb{R}} B^{\mathrm{sm}} = 1/2\dim_{\mathbb{R}} \mathcal{X}_s$, and $B \setminus B^{\mathrm{sm}}$ is of Hausdorff codimension 2. Moreover, B^{sm} is a Riemannian manifold such that the metric tensor is locally a Hessian (in the affine coordinates) of a potential function, $g_{ij} = \partial^2 F/\partial x_i \partial x_j$, and the potential satisfies the real Monge-Ampere equation $\det(\partial^2 F/\partial x_i \partial x_j) \equiv \mathrm{const}$.

Conjecture B (Kontsevich and Soibelman) The non-Archimedean space X admits a continuous surjective map to $\mathrm{Sk}(X)$ with the fibre non-Archimedean torus over an open dense subset of $\mathrm{Sk}(X)$ with complement of codimension 2; this fibration induces a singular affine structure on $\mathrm{Sk}(X)$.

Conjecture C (Kontsevich and Soibelman) The two spaces are naturally isomorphic as (singular) affine manifolds.

In [Sus18a] I have considered the following toy version of Conjecture C: I consider maximally unipotent degenerations of complex curves $\mathcal{X}_s$ of genus ≥ 1 endowed with flat (pseudo-)Kähler metrics with the Kähler form $i/2\Omega_s \wedge \bar{\Omega}_s$ where Ω is a relative holomorphic 1-form.

Let $\mathscr{X}$ be a model of X, i.e. a flat R-scheme such that $\mathscr{X} \otimes_R K \cong X$. Assume that the central fibre $\mathscr{X}_0$ is strictly normal crossings (such models are called *snc models*), then the dual intersection complex naturally embeds into X^{an}, its image is called the *skeleton* $\mathrm{Sk}(\mathscr{X})$ associated to the snc model $\mathscr{X}$. For a holomorphic differential Ω on X Kontsevich and Soibelman have defined a so-called *weight function* $\mathrm{wt}_\Omega : X^{\mathrm{an}} \to \mathbb{R}$. Mustaţa and Nicaise [MN15] have studied this function and have shown that it is piecewise affine $\mathrm{Sk}(\mathscr{X})$ for any snc model $\mathscr{X}$ with respect to the natural PL structure. Moreover, Nicaise and Xu [NX$^+$16b] have shown that the minimum of wt_Ω is related to the log canonical threshold. For $x, y \in X^{\mathrm{an}}$ define $x \sim y$ if and only if there exists a path from x to y that intersects the minimality locus of wt_Ω in finitely many points.

Theorem (S. [Sus18a]). The Gromov-Hausdorff limit of $(\mathcal{X}_s, \tilde{\omega}_s)$ is the metric graph $\mathrm{Sk}(\mathscr{X})/\sim$ where $\mathscr{X}$ is any snc model, with the length of an edge of $\mathrm{Sk}(\mathscr{X})$ given by the absolute value of the leading coefficient of the power series expansion of the form Ω in the local coordinate associated to the edge.

The solution uses an approach, due to Abraham Robinson, to view an algebraically closed valued field as an asymptotic limit of real closed fields; it allows one to treat elements of the field with strictly positive valuation as infinitesimals and to systematically relate the asymptotic behaviour of functions on $\mathcal{X}_s$ as s tends to 0 to non-Archimedean norms of functions on X^{an}. Robinson's asymptotic field has been used in a similar manner by Kramer and Tent [KT02] to compute asymptotic cones of Lie groups.

I aim to generalize the theorem above to higher dimensions using the theory of real differential forms of Chambert-Loir and Ducros [CLD12], [Gub16]. This theory defines sheaves of (p, q)-differential forms on X^{an} as objects that are given locally on so-called tropical charts. The latter are the data of an open set $U = \mathrm{trop}^{-1}(\mathrm{trop}(U))$ and a locally closed embedding of U into an algebraic torus. A differential form on a tropical chart U is given by a formal expression involving formal differentials d' and d'', of bidegree $(1, 0)$ and $(0, 1)$, respectively, of functions $\log|x_1|, \ldots, \log|x_n|$ where $x_1, \ldots, x_n$ are coordinate functions on the torus.

The support of a positive symmetric $(1, 1)$ form is a polyhedral space and is naturally endowed with a generalized Riemannian structure. I dispose of a tentative definition of the *non-Archimedean limit* of a degenerating family of real differential forms on $\mathcal{X}_s$ with values in the (p, q)-forms on X^{an} which makes precise the remark of Chambert-Loir and Ducros that the forms $d' \log|x|$ and $d'' \log|x|$ in their theory should be regarded as analogues of real forms $d \log_{|s|}|x|$ and $d \operatorname{Arg} x$ on a family of complex manifolds as $s \to 0$.

The collapsed limits of Conjecture A have been studied so far only in the hyperkähler case (see [GW00],[Tos10] [GTZ16], [TZ17]) by reducing them to the so-called large Kähler structure limits, where a complex manifold X is fixed and is assumed to be a holomorphic Lagrangian fibration over a complex manifold of dimension $1/2 \dim X$, the Kähler form tends towards a boundary of the Kähler cone and the volume of the fibres tends to 0. Unfortunately, this approach does not generalize to odd dimensions.

My **next goal** in this project is to prove the following statement, which is a natural generalization of the description of the Gromov-Hausdorff limit in the case of flat metrics on curves mentioned above:

Claim. Let α be the non-Archimedean limit of Kähler forms $\tilde{\omega}_s$ as $s \to 0$. Then the Gromov-Hausdorff limit of $(\mathcal{X}_s, \tilde{\omega}_s)$ is isometric (after rescaling so that the diameter is 1) to $\operatorname{supp}\alpha/\sim$ where for all $x, y \in X^{\mathrm{an}}$, $x \sim y$ if and only if there exists a path γ connecting x and y such that $\gamma \cap \operatorname{supp}\alpha$ is finite.

Establishing this theorem will provide a natural framework for attacking Conjecture C and Conjecture A in the odd-dimensional case (of course, showing that non-Archimedean limits of Kähler forms of Ricci-flat metrics satisfy the real Monge-Ampere equation remains a challenging analytic question).

2b. Singular affine structure on the essential skeleton.

If X is a variety with trivial canonical class then the essential skeleton $\mathrm{Sk}(X)$ is the minimality locus of the weight function wt_Ω for any $\Omega \in H^0(X, \Omega^n_X)$. Nicaise and Xu [NX16a] have shown that $\mathrm{Sk}(X)$ coincides with the skeleton $\mathrm{Sk}(\mathscr{X})$ of a particular kind of model, a *minimal dlt model*. Although such models are not canonical, $\mathrm{Sk}(\mathscr{X})$ is canonical as a topological space as $\mathscr{X}$ ranges over minimal dlt models. Nicaise, Xu and Yue Yu [NXY18] have found that any minimal dlt model $\mathscr{X}$ defines a surjective map

$p_{\mathscr{X}} : X \to B = \mathrm{Sk}(X)$ which is a fibration in non-Archimedean tori over a subset $B^{\mathrm{sm}} \subset B$ such that $B \setminus B^{\mathrm{sm}}$ is of codimension 2, as predicted by **Conjecture B**. The choice of a model $\mathscr{X}$ corresponds to the choice of a polyhedral decomposition of B and the choice of the fibration $p_{\mathscr{X}}$.

Let $B = \mathrm{Sk}(X)$ and let $p_{\mathscr{X}} : X^{\mathrm{an}} \to B$ be the retraction map associated to some minimal dlt model. Leaving aside the definition of the pullback $p^*_{\mathscr{X}}$ for the moment, let us formally write $\eta = p^*_{\mathscr{X}}\eta|_B$ for $\eta \in \mathcal{A}^{p,q}(X)$ if the tropical coordinates $-\log|x_1|, \dots, -\log|x_n|$ of all tropical charts used to define η are constant along the fibres of $p_{\mathscr{X}}$. Let α be the non-Archimedean limit of Kähler forms ω_s. If α is a pullback of its restriction to B and is strictly positive on B then it follows from the **Claim** (Project 2a) that the Gromov-Hausdorff limit of $(\mathcal{X}, \omega_s)$ is homeomorphic to B with the metric structure induced by α.

The Kähler forms of Ricci-flat metrics in a Kähler class $[\omega_0]$ are solutions to the complex Monge-Ampere equation $(\omega_0 + dd^c\varphi)^n = e^f\omega_0^n$ where $f = \log \dfrac{\Omega \wedge \bar{\Omega}}{\omega_0^n}$ for some holomorphic volume form Ω. The existence of the solution to the non-Archimedean Monge-Ampere equation has been proved by Boucksom, Favre and Jonsson[BFJ15]. I will make the following working assumption:

Assumption. Let α_0 be the non-Archimedean limit of $(\omega_0)_s$, and μ be the non-Archimedean limit of $(\Omega \wedge \bar{\Omega})_s$ as s tends to 0. Let φ_s be the solutions to the complex Monge-Ampere equations above, with the reference Kähler forms $(\omega_0)_s$, for s in a neighbourhood of 0. Then the non-Archimedean limit ψ of φ_s satisfies the non-Archimedean Monge-Ampere equation

$$(\alpha_0 + d'd''\psi)^n = \mu$$

If $\beta = \alpha_0 + d'd''\psi$ then it follows immediately from the definitions of differentials d', d'' that if $p : X^{\mathrm{sm}} \to B^{\mathrm{sm}}$ is a torus fibration and if $\beta|_{X^{\mathrm{sm}}} = p^*(\beta|_{B^{\mathrm{sm}}})$ then $\beta|_{B^{\mathrm{sm}}}$ is a solution to the real Monge-Ampere equation on B^{sm} in the affine coordinates induced by $p_{\mathscr{X}}$. Now, taking the **Assumption** into account, from the statement of Conjecture A we have that β belongs to the first Chern class of L in the non-Archimedean Dolbeault cohomology of X^{an} (which can be defined thanks to Chambert-Loir-Ducros theory).

My **next goal** in this project is to answer the following question:

Question. Given the first Chern class $c_1(L) \in H^{1,1}_{d''}(X)$ find all minimal dlt models $\mathscr{X}$ such that $c_1(L) = p^*_{\mathscr{X}}\eta$ for some $\eta \in H^{1,1}_{d''}(\mathrm{Sk}(X))$.

2c. Hodge theory for differential forms on non-Archimedean analytic spaces.

As has been remarked above, a $(1,1)$-form in the sense of Chamber-Loir-Ducros defines a generalized Riemannian structure on its support. It comes as no surprise then that it is possible to define a natural L^2 metric on the space of differential forms on X^{an}, and it is natural to ask if there is a non-Archimedean analogue of the Kähler condition and Hodge theory.

The notion of a *Kähler superform* on $\mathbb{R}^n$ has been introduced and studied by Lagerberg [Lag11] with certain parts of the complex theory, such as Kähler identities, the local dd^c-lemma and Bochner-Kodaira-Nakano idenity, carried over to the setting of superforms. The natural Dolbeault complex of (p,q)-forms on non-Archimedean analtyic spaces has been studied by Jell, in particular, a version of the Poincaré lemma holds for d' (and d'') and therefore $H^{p,0}_{d''}(X^{\mathrm{an}})$ is isomorphic to the p-the singular cohomology group of X^{an}. Furthermore, Jell, Shaw and Smacka[JSS15] have shown that the Dolbeault cohomology groups on a polyhedral space are naturally isomorphic to the tropical cohomology groups defined by Itenberg, Katzarkov, Mikhalkin and Zharkov [IKMZ16].

There are certain technical peculiarities in the non-Archimedean case, for example, the support of a typical Kähler form ω is a proper closed subset of X^{an} and therefore the theory should be developed for forms with support contained in $\mathrm{supp}\,\omega$ and under the assumption that $\mathrm{supp}\,\omega$ contains a skeleton of an snc model (to which X^{an} is homotopy equivalent), but otherwise the theory bears many similarities to the complex case.

My **goals** in this project are to prove non-Archimedean analogues of the main statements of Hodge theory on Kähler manifolds and its consequences:

- I expect that every Dolbeault cohomology class in $H^{p,q}_{d''}(X)$ has a unique harmonic representative;
- I expect the global dd^c-lemma, and non-Archimedean versions of the Kodaira vanishing theorem and the Kodaira embedding theorem to hold.

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